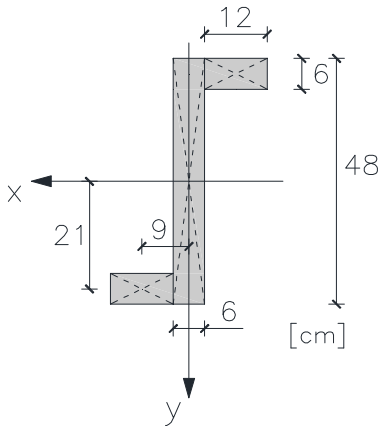
Zadanie 1



$$I_x = 2 \cdot \left(\frac{12 \cdot 6^3}{12} + 12 \cdot 6 \cdot 21^2\right) + \frac{6 \cdot 48^3}{12} = 119232 cm^4$$

$$I_y = 2 \cdot \left(\frac{6 \cdot 12^3}{12} + 12 \cdot 6 \cdot 9^2\right) + \frac{48 \cdot 6^3}{12} = 14256 cm^4$$

$$I_{xy} = 12 \cdot 6 \cdot 21 \cdot 9 + 12 \cdot 6 \cdot (-21) \cdot (-9) = 27216 cm^4$$

$$I_{1,2} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \frac{119232 + 14256}{2} \pm \sqrt{\left(\frac{119232 - 14256}{2}\right)^2 + 27216^2} = 66744 \pm 59124.452 \; [cm^4]$$

$$I_1 = 125868.452 cm^4$$

$$I_2 = 7619.548 cm^4$$

$$tg2\varphi_0 = \frac{2I_{xy}}{I_x - I_y} = \frac{2 \cdot 27216}{119232 - 14256} = 0.519$$

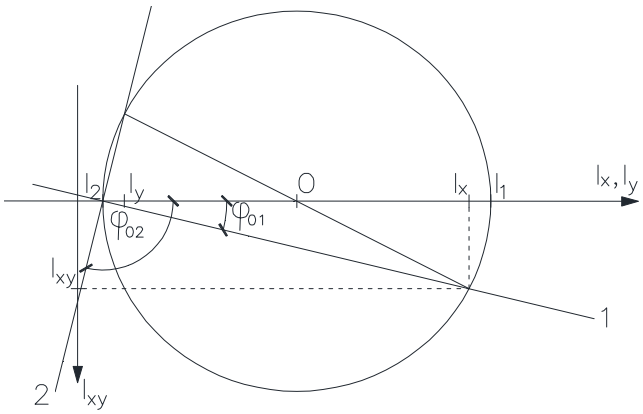
$$2\varphi_0 = arctg0.519 = 27.42°$$

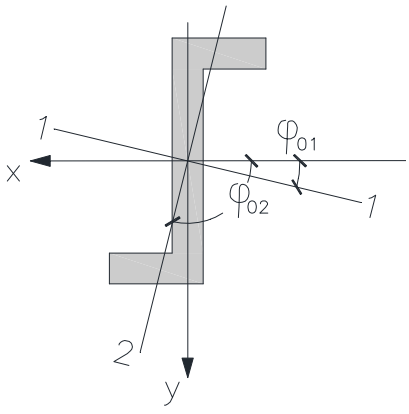
$$\varphi_0 = 13.71° = 13°42'$$

$$\underset{(+)}{(I_x - I_y)} \underset{(+)}{cos2\varphi_0} > 0 \Rightarrow max$$

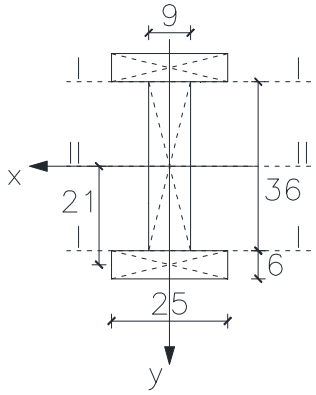
$$\varphi_{01} = 13.71° = 13°42'$$

$$\varphi_{02} = 103.71° = 103°42'$$





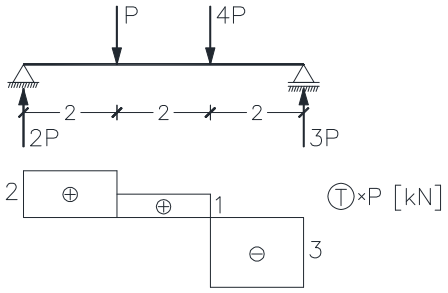
Zadanie 2



$$I_x = 2 \cdot \left(\frac{25 \cdot 6^3}{12} + 25 \cdot 6 \cdot 21^2\right) + \frac{9 \cdot 36^3}{12} = 168192 cm^4$$

$$S_x^I = 25 \cdot 6 \cdot 21 = 3150 cm^3$$

$$S_x^{II} = 25 \cdot 6 \cdot 21 + 18 \cdot 9 \cdot 9 = 4608 cm^3$$



$$T_y = |T_{max}| = 3P$$

Część I

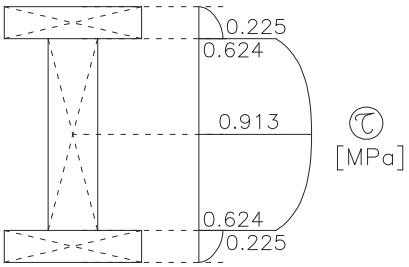
$P = 10kN \Longrightarrow T_y = 30kN$

$$\tau = \frac{T_y \cdot S_x}{I_x \cdot b}$$

$$\tau_I^g = \frac{30 \cdot 3150}{168192 \cdot 25} = 0.225 MPa$$

$$\tau_I^d = \frac{30 \cdot 3150}{168192 \cdot 9} = 0.624 MPa$$

$$\tau_{II} = \frac{30 \cdot 4608}{168192 \cdot 9} = 0.913 MPa$$



Część II

a)

$$t = \frac{T_y \cdot S_x^I}{I_x} = \frac{3P \cdot 3150}{168192} = 0.0562P \left[\frac{kN}{cm}\right]$$

$$P' = t \cdot e = 0.0562P \cdot 20 = 1.124P \, [kN]$$

$$N' = A \cdot R_\tau = \frac{\pi \cdot d^2}{4} \cdot R_\tau = \frac{\pi \cdot 2^2}{4} \cdot 10 = 31.416 \, [kN]$$

$$P' \leq N'$$

$$1.124P \leq 31.416$$

$$P \leq 27.950 kN$$

b)

$$\tau = \frac{T_y \cdot S_x}{I_x \cdot b} = \frac{3P \cdot 3150}{168192 \cdot 9} = 0.0624P \, [MPa]$$

$$\tau \leq R_\tau$$

$$0.0624P \leq 2$$

$$P \leq 32.051 kN$$